

Integrability of D1-brane on Group Manifold with Mixed Three Form Flux

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Abstract

We consider D1-brane as a natural probe of the group manifold with mixed three form fluxes. We determine Lax connection for given theory. Then we switch to the canonical analysis and calculate the Poisson brackets between spatial components of Lax connections and we argue for integrability of given theory.

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1 Introduction and Summary

The integrability in the AdS/CFT correspondence is fundamental for the calculations beyond perturbative theory. Famous example is the duality between $\mathcal{N} = 4$ super-Yang-Mills theory in four dimensions and Type IIB theory on $AdS_5 \times S^5$ with the Ramond-Ramond (RR) flux, where the exact string spectrum and the spectrum of anomalous dimensions in the SYM theory can be described by Bethe-ansatz equations². The integrability on the string theory side of the correspondence is based on the existence of the Lax connection that implies an existence of infinite number of conserved charges [5]. However this is only necessary condition since the integrability of the theory also requires that these conserved charges are in involution as was stressed in [6].

It is well known that integrability can be applied for group manifolds with non-trivial RR and NSNS fluxes. Such a famous example is string theory on AdS_3 background with non-trivial RR and NSNS fluxes. It turns out that in case of pure NSNS flux the string theory can be quantized using world-sheet conformal field theory techniques [31, 8, 9, 10, 11, 12]. On the other hand the RR AdS_3 backgrounds have more complicated CFT description [13] while these backgrounds are integrable as well [14, 15].

On the other hand the case of mixed RR/NSNS AdS_3 background is much more challenging either from CFT perspective or from the integrability point of view. One possibility is to consider small derivations from the pure NSNS point using conformal perturbative theory [16]. Another possibility was suggested in [17], where the starting point was pure RR background with new WZ term that represents the coupling to the NSNS flux. This beautiful construction leads to the rapid progress in understanding of the role integrability in theory with mixed fluxes, for related works, see [18, 19, 20, 21, 22, 23].

It is well known that the AdS_3 backgrounds with different fluxes are related by U-duality transformations. For example, type IIB S-duality relates $AdS_3 \times S^3 \times T^4$ backgrounds supported by different three-form fluxes: the pure RR flux background arises as the near horizon limit of D1-D5-brane system while background supported by mixed three form flux involves the near horizon limit of NS5-branes and fundamental strings in addition to the D1 and D5-branes. At the same time fundamental string transforms under S-duality to the bound state of D1-brane and fundamental string. Then one can ask the question whether D1-brane could be considered as another probe in string theory that naturally incorporates the coupling between NSNS and RR fields. In fact, the low energy descriptions of D1-brane is given by Dirac-Born-Infeld action together with Chern-Simons term with explicit coupling to RR and NSNS two forms. We demonstrated in our previous paper [24] that D1-brane on the group manifold with non-trivial NSNS flux is integrable. In this paper we extend given analysis to the most general background including dilaton, Ramond-Ramond zero form $C^{(0)}$ and Ramond-Ramond two form $C^{(2)}$ together with the three forms $F = dC^{(2)}$, $H = dB$ that can be expressed using the structure constants of

²For review, see [1, 2, 3, 4].

the group that defines the group manifold on which D1-brane propagates. We find that this D1-brane is integrable on condition when dilaton and Ramond-Ramond zero form are constants. Then we perform canonical analysis of given theory and calculate the Poisson brackets between spatial components of Lax connections. We show that this Poisson bracket has the form that ensures that the conserved charges are in involutions up to the well known problems with terms containing derivative of delta functions that need special regularizations [25, 26, 27]. Then we consider concrete example which is D1-brane on $AdS_3 \times S^3$ background with mixed RR/NS flux. We firstly show that the equation of motion for this D1-brane can be expressed as the equation of conservation of specific current which is however non-linear due to the specific form of D-brane action. Then introducing an auxiliary metric and corresponding constraint we can rewrite this current to the manifestly linear form³. Then fixing the gauge and for certain backgrounds we can find currents whose conservation law is special and that is an analogue of the holomorphic and anti-holomorphic currents in Wess-Zumino-Witten model [29]. Explicitly, we find that this occurs in case of D1-brane in the near horizon limit of D1-D5-brane background with zero electric flux. Surprisingly we also find that the same situation occurs in case of the background with non-zero RR and NSNS fluxes that arises from D1-D5-brane background through specific $SL(2, Z)$ transformation. This is very interesting result that suggests the possibility that for these values of fluxes D1-brane theory can be treated with the help of powerful techniques of two dimensional conformal field theory.

Let us outline our results. We show that D1-brane can be considered as a natural probe of backgrounds with mixed flux. We mean that given idea is very attractive and should be elaborated further. For example, it would be nice to explicitly determine world-sheet S-matrix for given theory in the AdS_3 background with mixed flux. It would be also nice to analyze classical solutions on the world-volume of given theory corresponding to possible magnon solutions and compare them with the string solutions. We hope to return to these problems in future. It would be also interesting try to extend given analysis to the supersymmetric D1-brane theory. Further question that deserves detailed treatment is the question of the conformal field theory description of D1-brane with electric flux on $AdS_3 \times S^3$ with specific values of fluxes. We hope to return to all these problems in future.

This paper is organized as follows. In the next section (2) we introduce D1-brane on the group manifold background with non-trivial NSNS and two RR forms. We analyze under which conditions is the world-sheet theory integrable. Then in section (3) we perform Hamiltonian analysis of given theory and calculate the Poisson brackets between spatial components of Lax connection. Finally in section (4) we consider D1-brane on various $AdS_3 \times S^3$ backgrounds with three form fluxes.

³This is similar situation as in the case of the equivalence between Nambu-Gotto and Polyakov action for bosonic string.

2 D1-brane on Group Manifold

In this section we introduce D1-brane action that governs the dynamics of D1-brane on general background. Recall that given action is the sum of DBI and CS term and has the form

$$\begin{aligned}
S &= -T_{D1} \int d\tau d\sigma e^{-\Phi} \sqrt{-\det \mathbf{A}} + \\
&+ T_{D1} \int d\tau d\sigma ((b_{\tau\sigma} + 2\pi\alpha' \mathcal{F}_{\tau\sigma}) C^{(0)} + c_{\tau\sigma}) , \\
\mathbf{A}_{\alpha\beta} &= G_{MN} \partial_\alpha x^M \partial_\beta x^N + 2\pi\alpha' \mathcal{F}_{\alpha\beta} + B_{MN} \partial_\alpha x^M \partial_\beta x^N , \\
\mathcal{F}_{\alpha\beta} &= \partial_\alpha A_\beta - \partial_\beta A_\alpha ,
\end{aligned} \tag{1}$$

where $x^M, M, N = 0, 1, \dots, D$ are embedding coordinates of D1-brane in the background that is specified by the metric $G_{MN}(X)$ and NSNS two form $B_{MN} = -B_{NM}$ together with Ramond-Ramond two form $C_{MN}^{(2)} = -C_{NM}^{(2)}$. We further consider background with non-trivial dilaton Φ and RR zero form $C^{(0)}$. Further, $\sigma^\alpha = (\tau, \sigma)$ are world-sheet coordinates of D1-brane and $b_{\tau\sigma}, c_{\tau\sigma}$ are pull-backs of B_{MN} and C_{MN} to the world-volume of D1-brane. Explicitly,

$$b_{\alpha\beta} \equiv B_{MN} \partial_\alpha x^M \partial_\beta x^N = -b_{\beta\alpha} , \quad c_{\tau\sigma} = C_{MN}^{(2)} \partial_\tau x^M \partial_\sigma x^N . \tag{2}$$

Finally $T_{D1} = \frac{1}{2\pi\alpha'}$ is D1-brane tension and $A_\alpha, \alpha = \tau, \sigma$ is two dimensional gauge field that propagates on the world-sheet of D1-brane.

It is useful to rewrite the action (1) into the form

$$\begin{aligned}
S &= -T_{D1} \int d\tau d\sigma e^{-\Phi} \sqrt{-\det g - (2\pi\alpha' \mathcal{F}_{\tau\sigma} + b_{\tau\sigma})^2} + \\
&+ T_{D1} \int d\tau d\sigma ((b_{\tau\sigma} + 2\pi\alpha' \mathcal{F}_{\tau\sigma}) C^{(0)} + c_{\tau\sigma}) ,
\end{aligned} \tag{3}$$

where $g_{\alpha\beta} = G_{MN} \partial_\alpha x^M \partial_\beta x^N, \det g = g_{\tau\tau} g_{\sigma\sigma} - (g_{\tau\sigma})^2$. From (3) we obtain the

equations of motion for x^M

$$\begin{aligned}
& \partial_M [\Phi] e^{-\Phi} \sqrt{-\det g - (2\pi\alpha' \mathcal{F}_{\tau\sigma} + b_{\tau\sigma})^2} - \\
& - \partial_\alpha \left[\frac{G_{MN} \partial_\beta x^N g^{\beta\alpha} \det g}{\sqrt{-\det g - (2\pi\alpha' \mathcal{F}_{\tau\sigma} + b_{\tau\sigma})^2}} \right] + \frac{\partial_M G_{KL} \partial_\alpha x^K \partial_\beta x^L g^{\beta\alpha} \det g}{2\sqrt{-\det g - (2\pi\alpha' \mathcal{F}_{\tau\sigma} + b_{\tau\sigma})^2}} + \\
& + \frac{(2\pi\alpha' \mathcal{F}_{\tau\sigma} + b_{\tau\sigma})}{\sqrt{-\det g - (2\pi\alpha' \mathcal{F}_{\tau\sigma} + b_{\tau\sigma})^2}} \partial_M B_{KL} \partial_\tau x^K \partial_\sigma x^L - \\
& - \partial_\tau \left[\frac{B_{MN} \partial_\sigma x^N (2\pi\alpha' \mathcal{F}_{\tau\sigma} + b_{\tau\sigma})}{\sqrt{-\det g - (2\pi\alpha' \mathcal{F}_{\tau\sigma} + b_{\tau\sigma})^2}} \right] + \partial_\sigma \left[\frac{B_{MN} \partial_\tau x^N (2\pi\alpha' \mathcal{F}_{\tau\sigma} + b_{\tau\sigma})}{\sqrt{-\det g - (2\pi\alpha' \mathcal{F}_{\tau\sigma} + b_{\tau\sigma})^2}} \right] + \\
& + \partial_M C^{(0)} (b_{\tau\sigma} + 2\pi\alpha' \mathcal{F}_{\tau\sigma}) + C^{(0)} \partial_M b_{KL} \partial_\tau x^K \partial_\sigma x^L - \\
& - \partial_\tau [C^{(0)} b_{MK} \partial_\sigma x^K] - \partial_\sigma [C^{(0)} b_{KM} \partial_\tau x^K] + \\
& + \partial_M C_{KL}^{(2)} \partial_\tau x^K \partial_\sigma x^L - \partial_\tau [C_{MK}^{(2)} \partial_\sigma x^K] - \partial_\sigma [C_{KM}^{(2)} \partial_\tau x^K] = 0
\end{aligned} \tag{4}$$

while the equations of motion for A_τ, A_σ take the form

$$\begin{aligned}
& \partial_\tau \left[e^{-\Phi} \frac{(2\pi\alpha' \mathcal{F}_{\tau\sigma} + b_{\tau\sigma})}{\sqrt{-\det g - (2\pi\alpha' \mathcal{F}_{\tau\sigma} + b_{\tau\sigma})^2}} + C^{(0)} \right] = 0 , \\
& \partial_\sigma \left[e^{-\Phi} \frac{(2\pi\alpha' \mathcal{F}_{\tau\sigma} + b_{\tau\sigma})}{\sqrt{-\det g - (2\pi\alpha' \mathcal{F}_{\tau\sigma} + b_{\tau\sigma})^2}} + C^{(0)} \right] = 0 .
\end{aligned} \tag{5}$$

Last two equations imply an existence of constant electric flux

$$\frac{e^{-\Phi} (2\pi\alpha' \mathcal{F}_{\tau\sigma} + b_{\tau\sigma})}{\sqrt{-\det g - (2\pi\alpha' \mathcal{F}_{\tau\sigma} + b_{\tau\sigma})^2}} + C^{(0)} = \Pi , \quad \Pi = \text{const} . \tag{6}$$

With the help of this constant we can express $2\pi\alpha' \mathcal{F}_{\tau\sigma} + b_{\tau\sigma}$ as

$$2\pi\alpha' \mathcal{F}_{\tau\sigma} + b_{\tau\sigma} = \frac{(\Pi - C^{(0)}) \sqrt{-\det g}}{\sqrt{e^{-2\Phi} + (\Pi - C^{(0)})^2}} , \tag{7}$$

so that the equations of motion (4) simplify considerably

$$\begin{aligned}
& - \partial_M [\sqrt{e^{-2\Phi} + (\Pi - C^{(0)})^2}] \sqrt{-\det g} + \\
& + \partial_\alpha \left[G_{MN} \partial_\beta x^N g^{\beta\alpha} \sqrt{-\det g} \sqrt{e^{-2\Phi} + (\Pi - C^{(0)})^2} \right] - \\
& - \frac{1}{2} \partial_M G_{KL} \partial_\alpha x^K \partial_\beta x^L \sqrt{-\det g} \sqrt{e^{-2\Phi} + (\Pi - C^{(0)})^2} + \\
& + \Pi H_{MKN} \partial_\tau x^K \partial_\sigma x^N + F_{MKN} \partial_\tau x^K \partial_\sigma x^N = 0 ,
\end{aligned} \tag{8}$$

where

$$\begin{aligned} H_{MNK} &= \partial_M B_{NK} + \partial_N B_{KM} + \partial_K B_{MN} , \\ F_{MNK} &= \partial_M C_{NK}^{(2)} + \partial_N C_{KM}^{(2)} + \partial_K C_{MN}^{(2)} . \end{aligned} \quad (9)$$

Now we are going to be more specific about the background. When we consider group manifold G we presume that the metric G_{MN} can be expressed as

$$G_{MN} = E_M^A E_N^B K_{AB} , \quad (10)$$

where for the group element $g \in G$ we have

$$J \equiv g^{-1} dg = E_M^A T_A dx^M , \quad (11)$$

where T_A is the basis of Lie Algebra \mathcal{G} of the group G . Note that $K_{AB} = \text{Tr}(T_A T_B)$. Further, from the definition (11) we obtain

$$dJ + J \wedge J = 0 \quad (12)$$

that implies an important relation

$$\partial_M E_N^A - \partial_N E_M^A + f_{BC}^A E_M^B E_N^C = 0 , \quad (13)$$

where

$$[T_B, T_C] = T_A f_{BC}^A . \quad (14)$$

In case of the fluxes F_{KLM} , H_{KLM} we presume following relations between them and the structure constants f_{ABC} of the Lie algebra \mathcal{G}

$$H_{MNK} E_A^M E_B^N E_C^K = \kappa f_{ABC} , \quad F_{MNK} E_A^M E_B^N E_C^K = \omega f_{ABC} , \quad (15)$$

where κ and ω are constants. The first one formula is well known relation that defines Wess-Zumino term when we describe motion of string on group space with B -flux ⁴. In case of Ramond-Ramond flux we introduce this relation in order to preserve symmetry between NS-NS and RR fluxes. At this place we will not discuss the problem whether background fields define a consistent string theory background and hence we can consider κ and ω as free parameters. On the other hand it is important to stress that when we discuss D1-brane on $AdS_3 \times S^3$ with mixed fluxes these coefficients κ and ω have concrete values in order to define consistent string theory background. We will discuss this case in more details in section (4).

With the help of (15) we can write

$$\begin{aligned} E_C^M H_{MKL} \partial_\tau x^K \partial_\sigma x^L &= \kappa f_{CAB} J_\tau^A J_\sigma^B \\ E_C^M F_{MKL} \partial_\tau x^K \partial_\sigma x^L &= \omega f_{CAB} J_\tau^A J_\sigma^B . \end{aligned} \quad (16)$$

⁴See for example [28] for nice discussion and calculations of Poisson brackets of various currents.

Note that E_A^M is inverse to E_M^B defined as

$$E_A^M E_M^B = \delta_A^B, \quad E_A^M E_N^A = \delta_N^M. \quad (17)$$

Now with the help of (13) and (16) we can rewrite the equations of motion (8) to the form that contains the current $J_\alpha^A = E_M^A \partial_\alpha x^M$

$$\begin{aligned} & - E_C^M \partial_M [\sqrt{e^{-2\Phi} + (\Pi - C^{(0)})^2}] \sqrt{-\det g} + \\ & + K_{CB} \partial_\alpha [J_\beta^B g^{\beta\alpha} \sqrt{-\det g} \sqrt{e^{-2\Phi} + (\Pi - C^{(0)})^2}] + \\ & + \Pi \kappa f_{CAB} J_\tau^A J_\sigma^B + \omega f_{CAB} J_\tau^A J_\sigma^B = 0. \end{aligned} \quad (18)$$

Now we are ready to analyze the integrability of given theory. Let us consider following current

$$\begin{aligned} L_\tau^A &= A J_\tau^A + B \sqrt{-g} g^{\sigma\alpha} \sqrt{e^{-2\Phi} + (\Pi - C^{(0)})^2} J_\alpha^A, \\ L_\sigma^A &= A J_\sigma^A - B \sqrt{-g} g^{\tau\alpha} \sqrt{e^{-2\Phi} + (\Pi - C^{(0)})^2} J_\alpha^A, \end{aligned} \quad (19)$$

where A and B are coefficients that will be determined by requirement that the current L_α^A is flat. First of all we calculate

$$\begin{aligned} \partial_\tau L_\sigma^A - \partial_\sigma L_\tau^A &= -A J_\tau^B J_\sigma^C f_{BC}^A + B(\Pi\kappa + \omega) f_{BC}^A J_\tau^B J_\sigma^C - \\ & - K^{AB} E_B^M \partial_M [\sqrt{e^{-2\Phi} + (\Pi - C^{(0)})^2}] \sqrt{-\det g}, \end{aligned} \quad (20)$$

where we used the equations of motion (18) together with the condition (13). As the next step we calculate

$$f_{BC}^A L_\tau^B L_\sigma^C = (A^2 - B^2 [e^{-2\Phi} + (\Pi - C^{(0)})^2]) f_{BC}^A J_\tau^B J_\sigma^C. \quad (21)$$

Collecting these two results together we obtain

$$\begin{aligned} & \partial_\tau L_\sigma^A - \partial_\sigma L_\tau^A + f_{BC}^A L_\tau^B L_\sigma^C = \\ & = (-A + B(\Pi\kappa + \omega) + A^2 - B^2 [e^{-2\Phi} + (\Pi - C^{(0)})^2]) f_{BC}^A J_\tau^B J_\sigma^C - \\ & - K^{AB} E_B^M \partial_M [\sqrt{e^{-2\Phi} + (\Pi - C^{(0)})^2}] \sqrt{-\det g}. \end{aligned} \quad (22)$$

Let us now discuss the result derived above. The expression on the second line is proportional to the currents while the expression on the third line contains derivatives of the background fields $C^{(0)}$ and Φ . Then clearly expressions on the second

and third line have to vanish separately in order L_α^A to be flat. The expression on the third line vanishes when we require that $\sqrt{e^{-2\Phi} + (\Pi - C^{(0)})^2}$ is constant. This can be ensured for non-zero electric flux Π when Φ and $C^{(0)}$ are constant. Then we have to demand that the expression on the second line in (22) vanishes. If we consider the ansatz $B = -\Lambda A$ we find the solutions in the form

$$\begin{aligned} A &= \frac{1}{1 - \Lambda^2(e^{-2\Phi} + (\Pi - C^{(0)})^2)}(1 + (\Pi\kappa + \omega)\Lambda) , \\ B &= -\frac{\Lambda}{1 - \Lambda^2(e^{-2\Phi} + (\Pi - C^{(0)})^2)}(1 + (\Pi\kappa + \omega)\Lambda) , \end{aligned} \quad (23)$$

where Λ is a spectral parameter. Finally we should mention that this is on-shell condition. On the other hand if we calculate the Poisson bracket between these currents we have to express A and B given in (23) using off-shell form of the combinations $e^{-2\Phi} + (\Pi - C^{(0)})^2$ and Π . Explicitly, from (7) we obtain

$$\begin{aligned} e^{-2\Phi} + (\Pi - C^{(0)})^2 &= \frac{e^{-2\Phi} \det g}{\det g + (2\pi\alpha' \mathcal{F}_{\tau\sigma} + b_{\tau\sigma})^2} , \\ \Pi &= \frac{e^{-\Phi}(2\pi\alpha' \mathcal{F}_{\tau\sigma} + b_{\tau\sigma})}{\sqrt{-\det g - (2\pi\alpha' \mathcal{F}_{\tau\sigma} + b_{\tau\sigma})^2}} + C^{(0)} . \end{aligned} \quad (24)$$

Inserting (23) and (24) into (19) we find off-shell formulation of flat current. In the next section we express the spatial components of the flat current using canonical variables and calculate Poisson bracket between them.

Let us summarize results derived in this section. We studied the dynamics of D1-brane on the group manifold with non-trivial NSNS and RR two form fluxes and together with dilaton and RR zero form. We argued that it is possible to define Lax connection for this theory and we showed that this Lax connection is flat on condition when the dilaton and RR zero form are constant. The existence of the Lax connection is a necessary condition of integrability. The additional condition is that corresponding conserved charges are in involution which can be seen from the form of the Poisson bracket between spatial components of Lax connection. The calculation of this Poisson bracket will be performed in the next section.

3 Poisson Brackets of Lax Connection

In this section we calculate the Poisson brackets between spatial components of Lax connection. To do this we have to develop the Hamiltonian formalism for $D1$ -brane action in general background. We start with the action (3) and find corresponding

conjugate momenta

$$\begin{aligned}
p_M &= \frac{\delta L}{\delta \partial_\tau x^M} = T_{D1} \frac{e^{-\Phi}}{\sqrt{-\det g - (2\pi\alpha' F_{\tau\sigma} + b_{\tau\sigma})^2}} (G_{MN} \partial_\alpha x^N g^{\alpha\tau} \det g + \\
&\quad + (2\pi\alpha' F_{\tau\sigma} + b_{\tau\sigma}) B_{MN} \partial_\sigma x^N) + T_{D1} (C^{(0)} B_{MN} \partial_\sigma x^N + C_{MN}^{(2)} \partial_\sigma x^N) , \\
\pi^\sigma &= \frac{\delta L}{\delta \partial_\tau A_\sigma} = \frac{e^{-\Phi} (2\pi\alpha' F_{\tau\sigma} + b_{\tau\sigma})}{\sqrt{-\det g - (2\pi\alpha' F_{\tau\sigma} + b_{\tau\sigma})^2}} + C^{(0)} , \quad \pi^\tau = \frac{\delta L}{\delta \partial_\tau A_\tau} \approx 0
\end{aligned} \tag{25}$$

and hence

$$\begin{aligned}
\Pi_M &\equiv p_M - \frac{\pi^\sigma}{(2\pi\alpha')} B_{MN} \partial_\sigma x^N - T_{D1} (C^{(0)} B_{MN} \partial_\sigma x^N + C_{MN}^{(2)} \partial_\sigma x^N) = \\
&= T_{D1} \frac{e^{-\Phi}}{\sqrt{-\det g - (2\pi\alpha' F_{\tau\sigma} + b_{\tau\sigma})^2}} G_{MN} \partial_\alpha x^N g^{\alpha\tau} \det g .
\end{aligned} \tag{26}$$

Using these relations it is easy to see that the bare Hamiltonian is equal to

$$H_B = \int d\sigma (p_M \partial_\tau x^M + \pi^\sigma \partial_\tau A_\sigma - \mathcal{L}) = \int d\sigma \pi^\sigma \partial_\sigma A_\tau \tag{27}$$

while we have three primary constraints

$$\begin{aligned}
\pi^\tau &\approx 0 , \quad \mathcal{H}_\sigma \equiv p_M \partial_\sigma x^M \approx 0 , \\
\mathcal{H}_\tau &\equiv \frac{1}{T_{D1}} \Pi_M G^{MN} \Pi_N + T_{D1} \left(e^{-2\Phi} + (\pi^\sigma - C^{(0)})^2 \right) g_{\sigma\sigma} \approx 0 .
\end{aligned} \tag{28}$$

Including these primary constraints to the definition of the Hamiltonian we obtain an extended Hamiltonian in the form

$$H = \int d\sigma (\lambda_\tau \mathcal{H}_\tau + \lambda_\sigma \mathcal{H}_\sigma - A_\tau \partial_\sigma \pi^\sigma + v_\tau \pi^\tau) , \tag{29}$$

where $\lambda_\tau, \lambda_\sigma, v_\tau$ are Lagrange multipliers corresponding to the primary constraints $\mathcal{H}_\tau \approx 0, \mathcal{H}_\sigma \approx 0, \pi^\tau \approx 0$. Now we have to check the stability of all constraints. The requirement of the preservation of the primary constraint $\pi^\tau \approx 0$ implies the secondary constraint

$$\mathcal{G} = \partial_\sigma \pi^\sigma \approx 0 . \tag{30}$$

In case of the constraints $\mathcal{H}_\tau, \mathcal{H}_\sigma$ we can easily show in the same way as in [24] that the constraints $\mathcal{H}_\tau, \mathcal{H}_\sigma$ are first class constraints and hence they preserved during the time evolution.

Now we are ready to proceed to the calculations of the Poisson brackets between spatial components of the flat current L_σ^A for different spectral parameters Λ and Γ

$$\{L_\sigma^A(\Lambda, \sigma), L_\sigma^B(\Gamma, \sigma')\} . \quad (31)$$

Recall that these are currents that define monodromy matrix and hence corresponding conserved charges. Using (24) and (25) we find that the spatial component of the current L_σ^A expressed using canonical variables has the form

$$L_\sigma^A = \frac{1 + \Lambda(\pi^\sigma \kappa + \omega)}{1 - \Lambda^2(e^{-2\Phi} + (\pi^\sigma - C^{(0)})^2)} \left(E_M^A \partial_\sigma x^M - \frac{\Lambda}{T_{D1}} E_B^M K^{AB} \Pi_M \right) . \quad (32)$$

In order to calculate (31) we need following Poisson brackets

$$\begin{aligned} \{x^M(\sigma), \Pi_N(\sigma')\} &= \delta_N^M \delta(\sigma - \sigma') , \\ \{E_M^A(\sigma), \Pi_N(\sigma')\} &= \partial_N E_M^A \delta(\sigma - \sigma') , \\ \{E_A^M(\sigma), \Pi_N(\sigma')\} &= \partial_N E_A^M \delta(\sigma - \sigma') , \end{aligned} \quad (33)$$

and also

$$\begin{aligned} &\{\Pi_M(\sigma), \Pi_N(\sigma')\} = \\ &= \frac{1}{2\pi\alpha'} (\pi^\sigma + C^{(0)}) H_{MNK} \partial_\sigma x^K \delta(\sigma - \sigma') + \frac{1}{2\pi\alpha'} F_{MNK} \partial_\sigma x^K \delta(\sigma - \sigma') + \frac{1}{2\pi\alpha'} \mathcal{G} B_{MN} \delta(\sigma - \sigma') \end{aligned} \quad (34)$$

and finally

$$\begin{aligned} &\{E_A^M \Pi_M(\sigma), E_B^N \Pi_N(\sigma')\} = -E_D^M f_{AB}^D \Pi_M \delta(\sigma - \sigma') + \\ &+ E_A^M \left(\frac{1}{2\pi\alpha'} (\pi^\sigma + C^{(0)}) H_{MNK} \partial_\sigma x^K + \frac{1}{2\pi\alpha'} F_{MNK} \partial_\sigma x^K + \frac{1}{2\pi\alpha'} \mathcal{G} B_{MN} \right) E_B^N \delta(\sigma - \sigma') . \end{aligned} \quad (35)$$

With the help of these results we obtain

$$\begin{aligned} &\{L_\sigma^A(\Lambda, \sigma), L_\sigma^B(\Gamma, \sigma')\} = \\ &= -\frac{1}{T_{D1}} f(\Lambda) f(\Gamma) (\Gamma + \Lambda) K^{AB} \partial_\sigma \delta(\sigma - \sigma') - \frac{1}{T_{D1}} K^{AB} \left[\Gamma \frac{df}{d\pi^\sigma}(\Lambda, \sigma) + \Lambda \frac{df}{d\pi^\sigma}(\Gamma, \sigma) \right] \mathcal{G} \delta(\sigma - \sigma') - \\ &- \frac{1}{T_{D1}} f(\Lambda) f(\Gamma) \left(\Lambda + \Gamma - \frac{1}{T_{D1}} \Lambda \Gamma [(\pi^\sigma + C^{(0)}) \kappa + \omega] \right) K^{AC} f_{CE}^B E_M^E \partial_\sigma x^M \delta(\sigma - \sigma') + \\ &+ \frac{\Lambda \Gamma}{T_{D1}^2} f(\Lambda) f(\Gamma) K^{AC} f_{CD}^B K^{DE} E_E^M \Pi_M \delta(\sigma - \sigma') , \end{aligned} \quad (36)$$

where we introduced function $f(\Lambda, \sigma)$

$$f(\Lambda, \sigma) = \frac{1 + \Lambda(\pi^\sigma(\sigma)\kappa + \omega)}{1 - \Lambda^2(e^{-2\Phi} + (\pi^\sigma(\sigma) - C^{(0)})^2)} \quad (37)$$

and used the fact that

$$\partial_\sigma f(\Lambda, \sigma) = \frac{df(\Lambda, \sigma)}{d\pi^\sigma} \partial_\sigma \pi^\sigma = \frac{df(\Lambda, \sigma)}{d\pi^\sigma} \mathcal{G} . \quad (38)$$

Now we demand that the expression proportional to δ function is equal to

$$\begin{aligned} - K^{AD} f_{DC}^B (X L_\sigma^C(\Lambda) - Y L_\sigma^C(\Gamma)) &= -K^{AD} f_{DC}^B (X f(\Lambda) - Y f(\Gamma)) E_M^C \partial_\sigma x^M + \\ + \frac{1}{T_{D1}} K^{AD} f_{DC}^B (A f(\Lambda) \Lambda - B f(\Gamma) \Gamma) E_E^M K^{CE} \Pi_M , \end{aligned} \quad (39)$$

where X and Y are unknown functions. Comparing (36) with (39) we derive following equations for X and Y

$$\begin{aligned} \frac{1}{T_{D1}} f(\Lambda) f(\Gamma) (\Lambda + \Gamma - \frac{1}{T_{D1}} \Lambda \Gamma ((\pi^\sigma + C^{(0)})\kappa + \omega)) &= X f(\Lambda) - Y f(\Gamma) , \\ \frac{\Lambda \Gamma}{T_{D1}} f(\Lambda) f(\Gamma) &= X f(\Lambda) \Lambda - Y f(\Gamma) \Gamma . \end{aligned} \quad (40)$$

These equations have following solutions

$$\begin{aligned} X &= \frac{\Lambda^2}{\Gamma - \Lambda} \frac{f(\Lambda)}{T_{D1}} [1 - \Gamma((\pi^\sigma + C^{(0)})\kappa + \omega)] , \\ Y &= \frac{\Gamma^2}{\Gamma - \Lambda} \frac{f(\Gamma)}{T_{D1}} [1 - \Lambda((\pi^\sigma + C^{(0)})\kappa + \omega)] \end{aligned} \quad (41)$$

that is generalization of the solutions found in [24] to the case of non-trivial Ramond-Ramond flux. In summary, we obtain final result

$$\begin{aligned} &\{L_\sigma^A(\Lambda, \sigma), L_\sigma^B(\Gamma, \sigma')\} = \\ &= -\frac{1}{T_{D1}} f(\Lambda) f(\Gamma) (\Gamma + \Lambda) K^{AB} \partial_\sigma \delta(\sigma - \sigma') - \frac{1}{T_{D1}} K^{AB} \left(\Gamma \frac{df(\Lambda)}{d\pi^\sigma} + \Lambda \frac{df(\Gamma)}{d\pi^\sigma} \right) \mathcal{G} \delta(\sigma - \sigma') - \\ &- \frac{1}{T_{D1}(\Gamma - \Lambda)} K^{AD} f_{DC}^B (\Gamma^2 f(\Gamma) [1 - \Lambda((\pi^\sigma + C^{(0)})\kappa + \omega)] L_\sigma^C(\Lambda) - \\ &- \Lambda^2 f(\Lambda) [1 - \Gamma((\pi^\sigma + C^{(0)})\kappa + \omega)] L_\sigma^C(\Gamma)) \delta(\sigma - \sigma') . \end{aligned} \quad (42)$$

We see that the expression proportional to $\mathcal{G} \approx 0$ vanishes on the constraint surface. We also see that there is still term proportional to the derivative of the delta function

that needs an appropriate regularization. Then the terms proportional to the delta functions are natural generalization of Poisson brackets of flat connection of principal chiral model with the Wess-Zumino term to the background with RR background two form. Note also that the form of the expression proportional to the delta functions implies that corresponding conservative charges are in involution which is the condition for the integrability of given theory [6].

4 Explicit Example: D1-Brane on $AdS_3 \times S^3$ with Three Form Fluxes

In this section we will analyze D1-brane on $AdS_3 \times S^3$ with three form fluxes. Before we proceed to the analysis of this specific background we still consider arbitrary group manifold with non-trivial fluxes but with constant dilaton and RR zero form. Then note that with the help of the flat condition we can rewrite the equation of motion into the form

$$\partial_\alpha \hat{J}^{A\alpha} = 0 , \quad (43)$$

where we introduced the current

$$\hat{J}^{A\alpha} = T_{D1} \left[\sqrt{e^{-2\Phi_0} + (C^{(0)} - \Pi)^2} \sqrt{-g} g^{\alpha\beta} J_\beta^A + (\Pi\kappa + \omega) \epsilon^{\alpha\beta} J_\beta^A \right] , \quad (44)$$

where $\epsilon^{\tau\sigma} = -\epsilon^{\sigma\tau} = 1$. We see that the current $\hat{J}^{A\alpha}$ is conserved. On the other hand we see that the current \hat{J}^A is non-linear and there is nothing more to say about. We can make given system more tractable when we introduce an auxiliary metric $\gamma_{\alpha\beta}$ that obeys the equation

$$T_{\alpha\beta} \equiv \frac{1}{2} \gamma_{\alpha\beta} \gamma^{\mu\nu} g_{\mu\nu} - g_{\alpha\beta} = 0 . \quad (45)$$

It is easy to see that this equation has solution $\gamma_{\alpha\beta} = g_{\alpha\beta}$. If we further introduce light-cone coordinates

$$\sigma^+ = \frac{1}{2}(\tau + \sigma) , \quad \sigma^- = \frac{1}{2}(\tau - \sigma) \quad (46)$$

we can rewrite the equation (43) into the form

$$\partial_+ \hat{J}^{A+} + \partial_- \hat{J}^{A-} = 0 , \quad \partial_\pm = \frac{\partial}{\partial \sigma^\pm} , \quad (47)$$

where

$$\begin{aligned}
\hat{J}^{A+} &= \frac{1}{2}(\hat{J}^{A\tau} + \hat{J}^{A\sigma}) = \\
&= \frac{T_{D1}}{2} \left[\sqrt{e^{-2\Phi_0} + (C^{(0)} - \Pi)^2} \sqrt{-\gamma} (\gamma^{\tau\alpha} J_\alpha^A + \gamma^{\sigma\alpha} J_\alpha^A) + (\Pi\kappa + \omega)(J_\sigma^A - J_\tau^A) \right] , \\
\hat{J}^{A-} &= \frac{1}{2}(\hat{J}^{A\tau} - \hat{J}^{A\sigma}) = \\
&= \frac{T_{D1}}{2} \left[\sqrt{e^{-2\Phi_0} + (C^{(0)} - \Pi)^2} \sqrt{-\gamma} (\gamma^{\tau\alpha} J_\alpha^A - \gamma^{\sigma\alpha} J_\alpha^A) + (\Pi\kappa + \omega)(J_\sigma^A + J_\tau^A) \right] .
\end{aligned} \tag{48}$$

As the next step we fix auxiliary metric to have the form $\gamma_{\alpha\beta} = \eta_{\alpha\beta}$, $\eta_{\mu\nu} = \text{diag}(-1, 1)$ keeping in mind that currents still have to obey the equation (45). In this gauge \hat{J}_\pm^A simplify considerably and we obtain

$$\begin{aligned}
\hat{J}^{A+} &= -\frac{1}{2}\hat{J}_-^A = \frac{T_{D1}}{2} \left[J_\sigma^A \left(\sqrt{e^{-2\Phi_0} + (C^{(0)} - \Pi)^2} + (\Pi\kappa + \omega) \right) \right. \\
&\quad \left. - J_\tau^A \left(\sqrt{e^{-2\Phi_0} + (C^{(0)} - \Pi)^2} + (\Pi\kappa + \omega) \right) \right] , \\
\hat{J}^{A-} &= -\frac{1}{2}\hat{J}_+^A = -\frac{T_{D1}}{2} \left[J_\tau^A \left(\sqrt{e^{-2\Phi_0} + (C^{(0)} - \Pi)^2} - (\Pi\kappa + \omega) \right) \right. \\
&\quad \left. + J_\sigma^A \left(\sqrt{e^{-2\Phi_0} + (C^{(0)} - \Pi)^2} - (\Pi\kappa + \omega) \right) \right] ,
\end{aligned} \tag{49}$$

where we introduced the light-cone metric with $\eta_{+-} = \eta_{-+} = -2$, $\eta^{+-} = \eta^{-+} = -\frac{1}{2}$ so that $\hat{J}^{A+} = \eta^{+-}\hat{J}_-^A = -\frac{1}{2}\hat{J}_-^A$, $\hat{J}^{A-} = \eta^{-+}\hat{J}_+^A = -\frac{1}{2}\hat{J}_+^A$. We see that for

$$\Pi\kappa + \omega = \sqrt{e^{-2\Phi_0} + (C^{(0)} - \Pi)^2} \tag{50}$$

the current \hat{J}_+^A vanishes identically and the equation (47) gives

$$\partial_+ \hat{J}_-^A = 0 , \quad \hat{J}_-^A = 2T_{D1} \sqrt{e^{-2\Phi_0} + (C^{(0)} - \Pi)^2} (J_\tau^A - J_\sigma^A) . \tag{51}$$

Note that we can write $\hat{J}_- = \hat{J}_-^A T_A = 2g^{-1}\partial_-g$. Then from (51) we obtain

$$\frac{1}{2}\partial_+ \hat{J}_- = -g^{-1}\partial_+ g g^{-1}\partial_- g + g^{-1}\partial_- \partial_+ g g^{-1}g = g^{-1}\partial_- [\partial_+ g g^{-1}]g = 0 \tag{52}$$

so that there is second current $\hat{J}_+ = \partial_+ g g^{-1}$ that obeys the equation

$$\partial_- \hat{J}_+ = 0 . \tag{53}$$

The equations (51) and (53) strongly resembles the conservations of currents in WZW model.

The previous analysis is valid for any group manifold with NSNS and RR fluxes and for constant dilaton and RR zero form. Now we would like to see whether the condition (50) can be realized in consistent string background. As the first case we consider $AdS_3 \times S^3 \times M$ background with pure RR flux where M is four torus T^4 of four-volume $V_M = (2\pi)^4 v \alpha'^2$ in the metric ds_M^2 that implies that each x^i are identified with the period $2\pi v^{1/4} \alpha'^{1/2}$. The background has the form [30]

$$\begin{aligned} ds^2 &= r_1 r_5 (ds_{AdS_3}^2 + ds_{S^3}^2) + \frac{r_1}{r_5} ds_M^2, \\ F &= \frac{2r_5^2}{g} (\epsilon + *_6 \epsilon_3), \\ e^{-\Phi} &= \frac{1}{g} \frac{r_5}{r_1}, \quad r_5 = \sqrt{g Q_5 \alpha'}, \quad r_1 = \frac{4\pi^2 \alpha'}{\sqrt{V_M}} \sqrt{g Q_1 \alpha'}, \end{aligned} \tag{54}$$

where $ds_{AdS_3}^2$ and $ds_{S^3}^2$ are line elements defined with the group elements from $SL(2, R)$ and $SU(2)$ respectively that define the currents J_α^A . Further, ds_M^2 is a Ricci-flat metric on M with volume V_M and where Q_1, Q_5 are the $D1$ - and $D5$ -brane charges. Finally ϵ is volume element of AdS_3 and $*_6 \epsilon$ is volume element of S^3 , where $*_6$ is Hodge dual in the six dimensions. Using (54) we obtain

$$\begin{aligned} \hat{J}_-^A &= -\frac{T_{D1} r_5^2}{g} \left[J_\sigma^A \left(\sqrt{1 + \Pi^2 \frac{g^2 r_1^2}{r_5^2}} + 1 \right) - J_\tau^A \left(\sqrt{1 + \Pi^2 \frac{g^2 r_1^2}{r_5^2}} + 1 \right) \right], \\ \hat{J}_+^A &= \frac{T_{D1} r_5^2}{g} \left[J_\sigma^A \left(\sqrt{1 + \Pi^2 \frac{g^2 r_1^2}{r_5^2}} - 1 \right) + J_\tau^A \left(\sqrt{1 + \Pi^2 \frac{g^2 r_1^2}{r_5^2}} - 1 \right) \right]. \end{aligned} \tag{55}$$

We see that \hat{J}_+^A vanishes identically in case when $\Pi = 0$ while \hat{J}_-^A is equal to

$$\hat{J}_-^A = \frac{Q_5}{\pi} (J_\tau^A - J_\sigma^A), \quad \partial_+ \hat{J}_-^A = 0. \tag{56}$$

This is expected result since in this case we have $D1$ -brane in the near horizon limit of $D1$ - $D5$ -brane system which is S-dual to the configuration of probe fundamental string in near horizon limit of the background NS-branes and fundamental strings. These models are known as WZW models [29] and can be analyzed using powerful conformal field techniques.

Let us now consider $D1$ -brane in this background. Recall that Type IIB theory has non-perturbative $SL(2, Z)$ symmetry

$$\begin{aligned} \hat{G}_{MN} &= e^{\frac{1}{2}(\hat{\Phi} - \Phi)} G_{MN}, \quad \hat{\tau} = \frac{a\tau + b}{c\tau + d}, \\ \hat{B}_{MN} &= cC_{MN}^{(2)} + dB_{MN}, \quad \hat{C}_{MN}^{(2)} = aC_{MN}^{(2)} + bB_{MN}, \end{aligned} \tag{57}$$

where $\tau = C^{(0)} + ie^{-\Phi}$ and where $ad - bc = 1$. Note that S-duality transformation corresponds to the following values of parameters $a = 0, b = 1, c = -1, d = 0$. Then we find that S-dual background has the form

$$\begin{aligned} e^{-2\hat{\Phi}} &= \frac{g^2 r_1^2}{r_5^2} = \frac{g^2 Q_1}{v Q_5} , \\ d\hat{s}^2 &= e^{-\Phi} ds^2 = \frac{1}{g} r_5^2 (ds_{AdS_3}^2 + ds_{S^3}^2) + g ds_M^2 = Q_5 \alpha' (ds_{AdS_3}^2 + ds_{S^3}^2) + g ds_M^2 , \\ H &= 2Q_5 \alpha' (\epsilon_3 + *_6 \epsilon_3) \end{aligned} \tag{58}$$

so that it is easy to see that the currents \hat{J}^A have the form

$$\begin{aligned} \hat{J}_-^A &= -T_{D1} \alpha' Q_5 \left[J_\sigma^A \left(\sqrt{\frac{g^2 Q_1}{v Q_5} + \Pi^2} + \Pi \right) - J_\tau^A \left(\sqrt{\frac{g^2 Q_1}{v Q_5} + \Pi^2} + \Pi \right) \right] , \\ \hat{J}_+^A &= T_{D1} \alpha' Q_5 \left[J_\sigma^A \left(\sqrt{\frac{g^2 Q_1}{v Q_5} + \Pi^2} - \Pi \right) + J_\tau^A \left(\sqrt{\frac{g^2 Q_1}{v Q_5} + \Pi^2} - \Pi \right) \right] . \end{aligned} \tag{59}$$

It is clear that \hat{J}_+^A does not vanish for finite values of parameters. On the other hand we easily see that \hat{J}_+^A vanishes identically when we consider the formal limit $g \rightarrow 0$. Physically this is the situation when D1-brane becomes infinite heavy and decouples so that the probe can be considered as the collection of Π fundamental strings. In this case the model corresponds to ΠQ_5 level WZW model that can be studied by conventional conformal field theory techniques. However it is important to stress that this is not possible in case of finite value of the string coupling constant.

Finally we consider more general case when we perform $SL(2, Z)$ duality transformation from the near horizon limit of D1-D5-brane background. We begin with the observation [31] that the near-horizon limit and S-duality commutes. Then for the general form of $SL(2, Z)$ transformation (with $C^0 = 0$) we obtain (using $\tau^* = -\tau$)

$$\begin{aligned} \hat{C}^{(0)} &= \frac{ace^{-2\Phi} + bd}{c^2 e^{-2\Phi} + d^2} , \quad e^{-\hat{\Phi}} = \frac{e^{-\Phi}}{c^2 e^{-2\Phi} + d^2} , \\ d\hat{s}^2 &= \sqrt{c^2 e^{-2\Phi} + d^2} ds^2 , \quad \hat{B} = cC^{(2)} , \quad \hat{C}^{(2)} = aC^{(2)} . \end{aligned} \tag{60}$$

Let us start with the symmetric flux background that corresponds following values of parameters a, b, c, d

$$a = 1 , c = 1 , b = 0 , d = 1 . \tag{61}$$

It turns out that in this case the current \hat{J}_+^A vanishes identically in case when $\Pi = 0$ while \hat{J}_-^A is equal to

$$\hat{J}_-^A = \frac{Q_5}{\pi} (J_\tau^A - J_\sigma^A) , \quad \partial_+ \hat{J}_-^A = 0 . \tag{62}$$

In fact, this remarkable result is valid whenever the parameter b is equal to zero. Explicitly, when $b = 0$ we find from the condition $ad - bc = 1$ that $a = d = 1$ that implies

$$\hat{C}^{(0)} = \frac{ce^{-2\Phi}}{c^2e^{-2\Phi} + d^2}, \quad e^{-\hat{\Phi}} = \frac{e^{-\Phi}}{c^2e^{-2\Phi} + d^2}. \quad (63)$$

Then for $\Pi = 0$ and for the background given above we obtain that the currents (49) have the form

$$\begin{aligned} \hat{J}_-^A &= -\frac{T_{D1}r_5^2}{g} (J_\sigma^A - J_\tau^A) \left(\frac{gr_1}{r_5} \sqrt{c^2e^{-2\Phi} + 1} \sqrt{\frac{e^{-2\Phi}}{c^2e^{-2\Phi} + 1}} + 1 \right), \\ \hat{J}_+^A &= \frac{T_{D1}r_5^2}{g} (J_\sigma^A + J_\tau^A) \left(\frac{gr_1}{r_5} \sqrt{c^2e^{-2\Phi} + 1} \sqrt{\frac{e^{-2\Phi}}{c^2e^{-2\Phi} + 1}} - 1 \right), \end{aligned} \quad (64)$$

where the first square root $\sqrt{c^2e^{-2\Phi} + 1}$ follows from the definition of dual line element (60) and the second one from the fact that $e^{-2\hat{\Phi}} + (C^{(0)})^2 = \frac{e^{-2\Phi}}{c^2e^{-2\Phi} + 1}$. We immediately see that the currents \hat{J}^A have the same form as in case of the original near horizon limit of D1-D5-brane background where \hat{J}_+^A vanishes identically.

Let us outline results derived in this section. We analyzed the conditions under which we can find holomorphic or antiholomorphic currents for D1-brane in the background $AdS_3 \times S^3$ with different combinations of NSNS and RR fluxes. While D1-brane is integrable for any values of fluxes and world-volume electric flux it possesses two holomorphic and anti-holomorphic currents that allow more powerful conformal field theory analysis in near horizon limit D1-D5-brane background on condition when the electric flux is zero. We also showed that this hold in case of $AdS_3 \times S^3$ with mixed fluxes that is related to the original D1-D5-brane system by $SL(2, Z)$ duality transformation.

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